Multiple Level Set Method for Optimal Design of Nonlinear Magnetostatic System

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This paper proposes a multiple level set method for multi-material shape optimization in the nonlinear magnetostatic system. The proposed method uses the sign combination of the level set functions to identify the different material regions and their interfaces. The velocity for the level set equations is coupled with the velocity field from the continuum sensitivity formula involving nonlinearity of the magnetic materials. The multiple material interfaces are simultaneously deformed by solving the multiple level set equations. A numerical example is tested to show usefulness of the proposed method.

***Index Terms*—Continuum sensitivity, finite element method, multiple level set method, nonlinear ferromagnetic, shape optimization.**

I. Introduction

THE LEVEL SET METHOD has been employed for many shape optimization problems thanks to its versatility in dealing with complicated geometry change. However, since the conventional level set method uses a single level set function to divide the material region, it can deform only one kind of the interface in the optimization problem. Meanwhile, the magnetic system usually consists of more than two materials such as the air, iron, current, or permanent magnet. Recently, the multiple level set method was reported for the multi-material magnetostatic system for the shape optimization [1]. However, the multiple level set method was applied for the linear magnetic system. Since most of the real magnetic systems are designed to be operated up to the magnetic saturation for full use of the ferromagnetic material, nonlinearity of the magnetic material should be taken into account for their optimal design.

A level set based optimization for the nonlinear magnetostatic system was tried in the former researches [2], but it was based on the discrete sensitivity analysis. Even though the discrete sensitivity analysis is one of good methods for sensitivity calculation, it has some problems and difficulties such as dependence on discretization model and resulting complexity in program implementation. On the other hand, since the continuum sensitivity analysis uses the closed form of sensitivity formula analytically derived from the variational governing equation, it can overcome the problems of the discrete approach. More importantly, it can be easily coupled with the level set method through the common velocity term. The velocity field from the continuum sensitivity formula well matches with the velocity field in the level set equation.

This paper proposes the multiple level set method for the nonlinear magnetostatic system with multiple material interfaces. The multiple material regions are identified by the sign combination of the level set functions. The velocity fields on the corresponding interfaces are calculated using the continuum sensitivity formula involving the magnetic nonlinearity and the multiple interfaces are simultaneously deformed by solving the multiple level set equations. An inductor shape design problem is tested using the proposed method to show its usefulness.

II. Multiple Level Set Method

When the conventional level set method is employed for the shape design problems, the material regions and their interface are identified using the sign of the level set function ϕ. The conventional method expresses the shape variation by solving a level set equation.

$$\frac{\partial \phi}{\partial t} + \mathbf{V} \cdot \nabla \phi = 0 \tag{1}$$

where t is the time and $\mathbf{V}$ is the velocity field. However, the single equation cannot be used to design more than two material system. On the other hand, the multiple level set method can design the multi-material system using the multiple level set functions. This method requires the corresponding number of the level set equations.

$$\frac{\partial \phi_i}{\partial t} + \mathbf{V}_i \cdot \nabla \phi_i = 0, \qquad \text{for} \qquad i = 1, 2, \cdots, m \tag{2}$$

The number of the equations can be minimized by using the sign combination of the level set functions to distinguish the regions. In this method, m level set functions are sufficient to represent upto 2^m materials. Fig. 1 shows a diagram that illustrates how to distinguish the material regions.

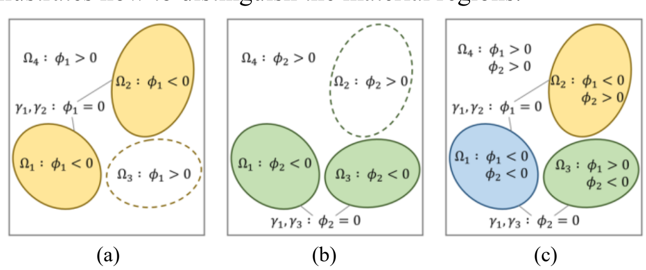

Fig. 1. Diagram of multiple level set method. (a) Single level set function ϕ_1. (b) Single level set function ϕ_2. (c) Multiple level set function ϕ_1 and ϕ_2.

III. Continuum Sensitivity Formula for Nonlinear Magnetostatic System

In the magnetostatic system, an objective function of the optimization problem is defined as a regional integral of the magnetic potential and the field as

$$\mathrm{F} = \int_{\Omega} g\left(\mathbf{A}, \mathbf{B}(\mathbf{A})\right) \mathrm{m}_p \, d\Omega \tag{3}$$

where g is the differentiable function, $\mathbf{A}$ is the magnetic vector potential, $\mathbf{B}()$ is the curl operator, and m_p is the characteristic function of the integration region. The continuum sensitivity formula for a nonlinear magnetostatic system can be derived using the material derivative concept of continuum mechanics and the adjoint variable method along, which are based on the Newton-Raphson algorithm for nonlinear analysis. The material derivative of the objective function gives

$$\begin{aligned} \dot{\mathrm{F}} = \int_{\Omega} \Big[& \mathbf{g_A}(\mathbf{A}) \cdot \dot{\mathbf{A}} + \mathbf{g_B}(\mathbf{A}) \cdot \mathbf{B}(\dot{\mathbf{A}}) \\ & - \mathbf{g_A}(\mathbf{A}) \cdot \left((\mathbf{V} \cdot \nabla) \mathbf{A} \right) - \mathbf{g_B}(\mathbf{A}) \cdot \mathbf{B}\left((\mathbf{V} \cdot \nabla) \mathbf{A} \right) \Big] \mathrm{m}_p \, d\Omega \end{aligned} \tag{4}$$

where $\mathbf{g_A}$ and $\mathbf{g_B}$ are the partial derivative of g with respect to the state and field variable, respectively. The adjoint variable $\boldsymbol{\lambda}$ is calculated by solving the adjoint equation shown as follows:

$$\nabla \times \mathbf{P}(\boldsymbol{\lambda}) = \left[\mathbf{g_A}(\mathbf{A}) + \nabla \times \mathbf{g_B}(\mathbf{A}) \right] \mathrm{m}_p \tag{5}$$

where $\mathbf{P}(\boldsymbol{\lambda}) \equiv \nu \mathbf{B}(\boldsymbol{\lambda}) + 2\kappa \mathbf{B}(\mathbf{A})^{\mathrm{T}} \mathbf{B}(\boldsymbol{\lambda}) \mathbf{B}(\mathbf{A})$ (6)

ν is the magnetic reluctivity and $\kappa = d\nu / dB^2$ [3]. The continuum sensitivity formula is obtained as a form of surface integral on the material interfaces as

$$\begin{aligned} \dot{\mathrm{F}} = \int_{\gamma} \Bigg[& (\nu_1 - \nu_2) \mathbf{B}(\boldsymbol{\lambda}_2) \cdot \left(\frac{2\kappa_2}{\nu_2} \mathbf{B}(\mathbf{A}_2) B_t(\mathbf{A}_2) B_t(\mathbf{A}_1) \right. \\ & \left. + \mathbf{B}(\mathbf{A}_1) \right) - (\mathbf{J}_1 - \mathbf{J}_2) \cdot \boldsymbol{\lambda}_2 \Bigg] \mathbf{V} \cdot \mathbf{n} \, d\Gamma \end{aligned} \tag{7}$$

where the subscripts 1 and 2 mean that the variables belong to the different region, the subscript t denotes the tangential component of the variable, $\mathbf{J}$ is the current density, and $\mathbf{n}$ is the unit normal vector. On the right-hand side of (7), the first, second, and third terms mean the sensitivity to the non-linearity, permeability, and current density, respectively. This formula enables the geometrical variation of not only the interface between the different permeability regions, but also the current surface.

IV. Numerical Test

Feasibility of the multiple level set method and the continuum sensitivity formula for a nonlinear magnetostatic system is shown by an inductor shape design problem.

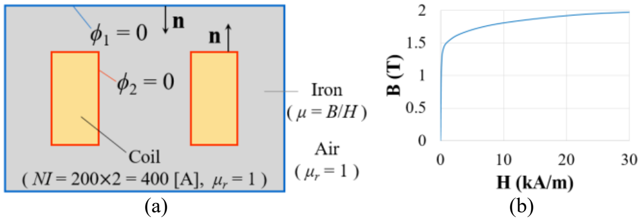

Fig. 2. (a) Initial state of inductor model. (b) B-H curve of the iron.

Fig. 2 shows the initial geometry of the inductor model and B-H curve of the iron. The zero level of level set functions ϕ_1 and ϕ_2 indicate each interfaces as shown in Fig. 2. The design is done under two equality constraints of given constant volumes of the iron and the coil of a constant current density. The design objective is to maximize the inductance of the system. Since the inductance is directly proportional to the system energy, the system energy $\mathrm{W_m}$ is taken as the objective function;

$$\mathrm{F} = \mathrm{W_m} = \int_{\Omega} \nu \left| \mathbf{B}(\mathbf{A}) \right|^2 d\Omega. \tag{8}$$

The adjoint variable for the objective function is equal to the state equation. The velocity field $\mathbf{V}_i$ in the equation (2) is determined by the continuum sensitivity formula (7) as follows:

$$\mathbf{V}_1 = (\nu_i - \nu_a) \mathbf{B}(\mathbf{A}_a) \cdot \mathbf{B}(\mathbf{A}_i) \mathbf{n} \tag{9}$$

$$\mathbf{V}_2 = \left[(\nu_c - \nu_a) \mathbf{B}(\mathbf{A}_a) \cdot \mathbf{B}(\mathbf{A}_c) - \mathbf{J}_c \cdot \mathbf{A}_c \right] \mathbf{n} \tag{10}$$

the subscripts of a, i, and c indicate that the variables belong to air, iron, and coil, respectively.

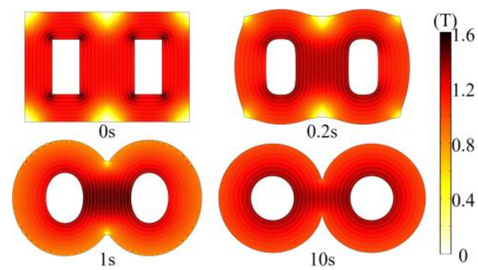

Fig. 3. Shape variation and flux distribution during optimization of inductor.

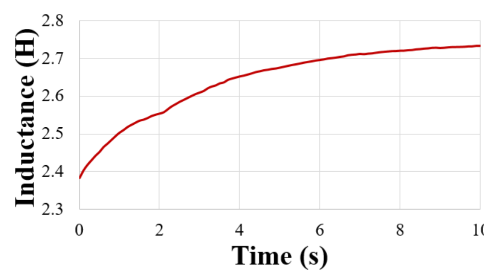

Fig. 4. Variation of inductance during optimization for inductor model.

Fig. 3 shows the evolution of the interfaces and the flux density distribution during the optimization. The cross-section of the iron is gradually deformed into two round ones in the final design. The final round shape of the iron prevents the local magnetic saturation and allows the minimum magnetic reluctance of the inductor. Fig. 4 shows the evolution of the objective function. The inductance of the final design is 14.7% greater than the initial state.

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